# *An O(n) Approximation for the Double Bounding Box Problem*

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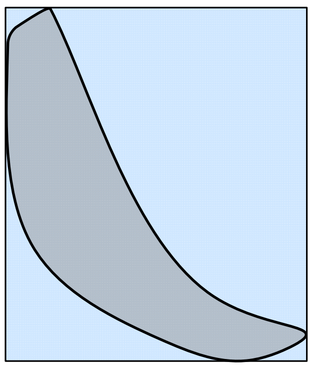
## Bounding Boxes

Bounding boxes approximate arbitrary 2D geometries

- computer graphics
- simulation, games
- spatial indexes



Here

- *axis-aligned* bounding boxes

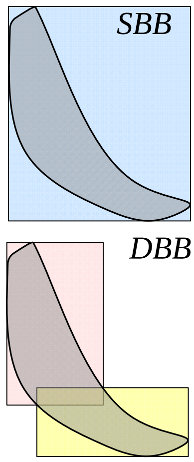
Benefits

- Easy to compute in O($n$)
- Simple, quick a-priori test for geometric conditions (e.g. 'is inside', 'overlaps') in O(1)

# Idea: Double Bounding Boxes

False hits require *exact* (costly) geometric checks. Our idea:

- We use *two* bounding boxes to better approximate a shape
- They may overlap
- They should have a minimal area
- Also $O(1)$ geometric a-priori checks



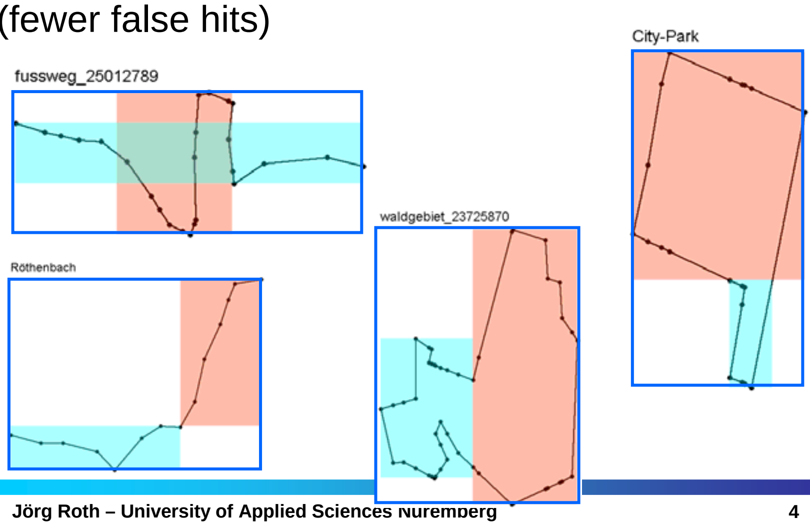

→ ***Double Bounding Boxes, DBB***

(we call the traditional one the *Single Bounding Box*, *SBB*)



# DBBs

DBBs approximate a shape much better (fewer false hits)

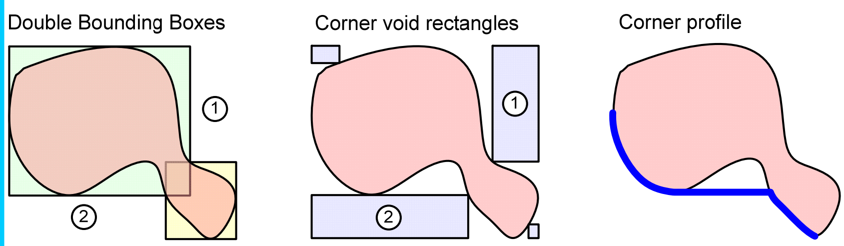


# Compute DBBs

How to compute *minimal* DBBs?

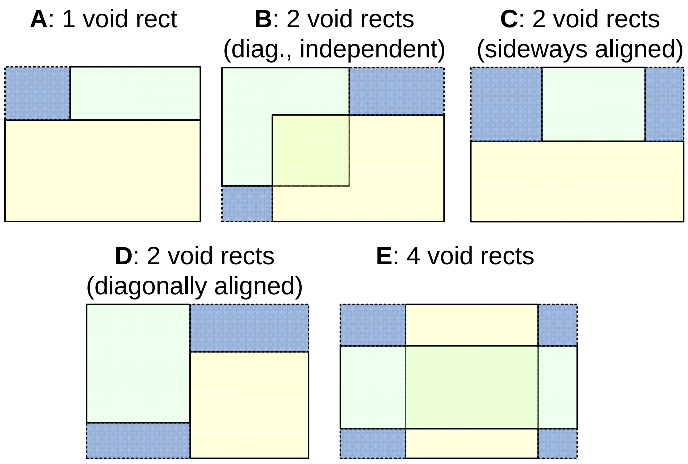
- We look at the inverse areas, the *void rects*
- A void rect's corner resides on a *corner profile*

→ We 'only' have to compute 4 corner profiles and select maximum void rects.





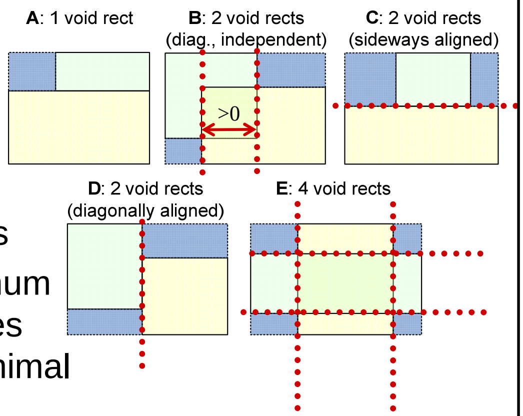

# DBB cases




Cases A-D can be rotated → total 15 sub-cases

## Compute DBBs

- Some cases have certain conditions. E.g. case E: all 4 void rects have to be aligned
- We compute the maximum area of all 15 sub-cases
- The overall maximum of the 15 sub-cases represents the minimal DBB






## Exact DBBs

There exists an algorithm that computes the minimal DBB:

- Publication:

  Jörg Roth: *The Approximation of Two-Dimensional Spatial Objects by Two Bounding Rectangles*
  Spatial Cognition & Computation: An Interdisciplinary Journal, Vol. 11, Issue 2, 2011, ISSN 1387-5868, 129-152
- The algorithm computes the *theoretical* minimum
- Requires $O(n \cdot \log n)$ steps for $n$ geometry points
- Reason: computing the corner profiles needs a kind of sorting

# Quick DBBs

Our new idea:

- We replace the $O(n \cdot \log n)$ algorithm to compute maximum void rects by an $O(n)$ approximation
- The rest of the algorithm remains unchanged, i.e.

```
iterate through all 15 sub-cases {
    generate the maximum void rectangle(s) that fulfil(s)
            the conditions related to this case;
    sum up the void rectangle areas;
    if (sum of areas > area of the previous best solution)
        store as the new best solution;
}
return the best solution;
```



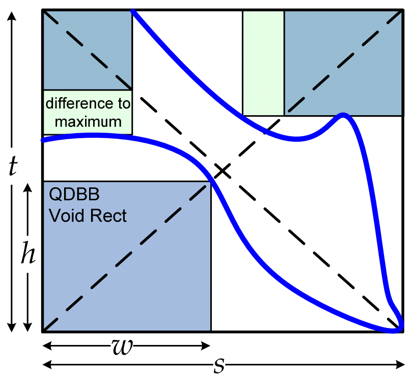
# Quick DBBs

Approximation for void rects:

- We do not consider maximum void rects, but only sub-maxima that have the SBB's aspect ratio, i.e.

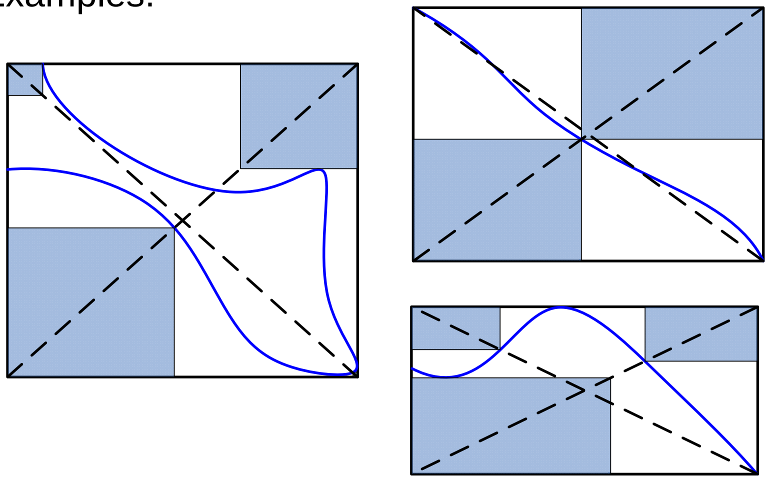
$$\frac{w}{h} = \frac{s}{t}$$

- Not the maximum, but easy to compute
- Note: with other aspect ratios the void rects often do not construct DBBs



## Quick DBBs

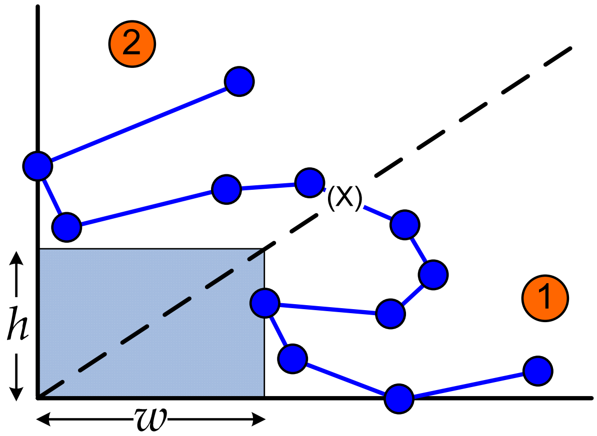
Examples:





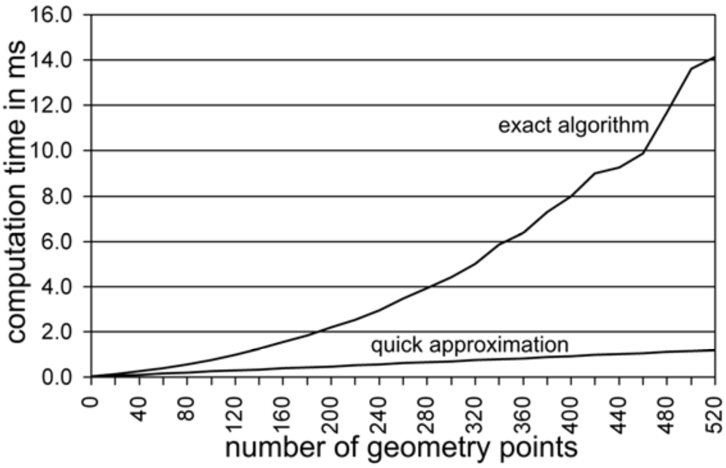
## Quick DBBs

Quick void rect construction:



# Runtime measurements

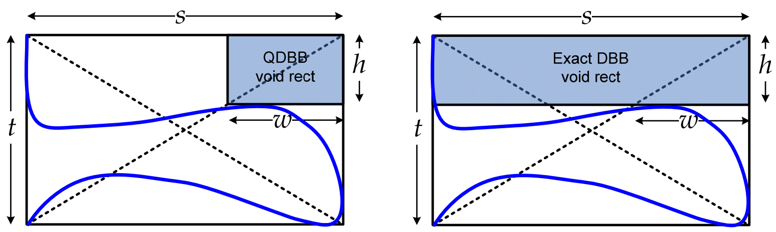
Comparison QDDB runtime to exact DBB






# QDBB worst case

Worst case considerations:



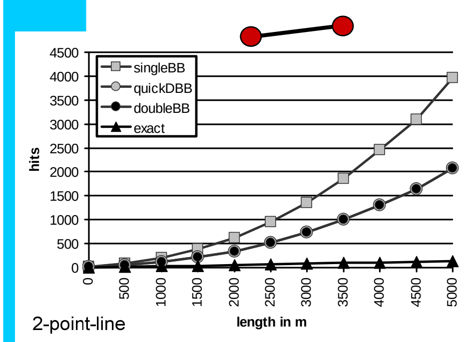

- Area ratio between QDBB and exact DBB:

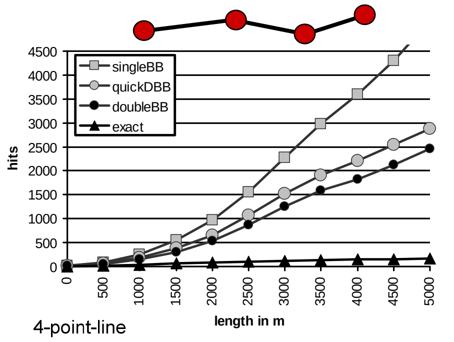
$$\frac{A_Q}{A_E} = \frac{s \cdot \left(t - h^2/t\right)}{s \cdot (t-h)} = \frac{h+t}{t}$$

- Boundary value is 2, i.e. in worst case the QDBB area has twice the size of the theoretical value

# Measurements with real data

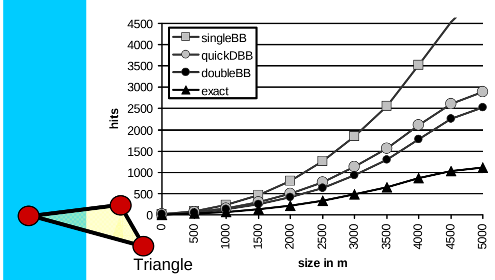
- 100,000 queries
- overlap test of random geometries with real geo data (Open Street Map)

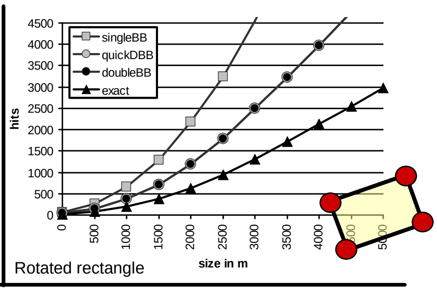


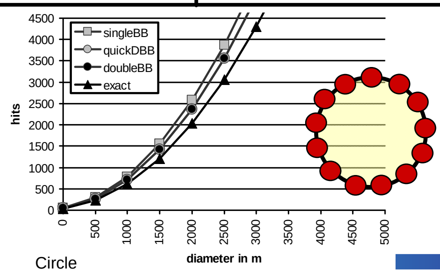






# Measurements with real data



# Measurements with real data

The difference between QDBB and exact DBB is small in reality:

- QDDB produce only 10.9% more false hits than the theoretically optimal DBB

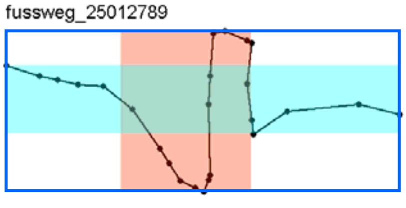
But:

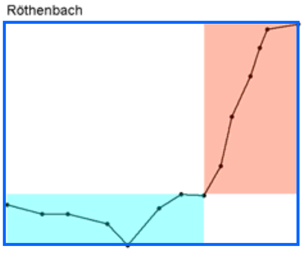
- The SBB produces 2.03 times more false hits than the QDBB!
- This means: SBBs produce twice as much (costly) exact geometric checks than QDBBs



# Summary

- DBBs are more suitable than SBBs to approximate real geometries
- The quick approximation only requires O(n) steps
- Worst case: 2 times larger areas
- Real data: nearly as good as the theoretical optimum



# Contact

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